ground water

Steady Flow to a Well Near a Stream with a Leaky Bed

by Mark Bakker[1] and Erik I. Anderson[2]

Abstract

We present an explicit analytic solution for steady, two-dimensional ground water flow to a well near a leaky streambed that penetrates the aquifer partially. Leakage from the stream is approximated as occurring along the centerline of the stream. The problem domain is infinite and pumping on one side of the stream induces flow on the other side. The solution includes the effects of uniform flow in the far field and a sloping hydraulic head in the stream. We use the solution to investigate the interaction between ground water and surface water in the stream, the effects of pumping on the opposite side of the stream, and the effects of the leaky streambed on the capture zone envelope of the well. We develop a relationship between parameters such that the pumping well will not capture water from the stream, or from the opposite side of the stream. When the discharge of the well is large enough to capture water from the stream, the shape of the capture zone envelope depends on flow conditions on the side of the stream opposite the well.

Introduction

Pumping of ground water near streams is of considerable interest in issues of water supply, water quality, and water rights administration. There is a long history of analytical studies focusing on the transient interaction between streams and aquifers due to pumping wells. Early investigators assumed that the streams were fully penetrating and that the streambeds were not clogged with low-permeability silts (Theis 1941; Glover and Balmer 1954). Investigators soon realized that clogging of the streambed has a major impact on the ground water flow field; ad hoc approaches using extended flow lengths were developed to deal with this complication (Kazman 1948; Walton 1963). Hantush (1965) presented an analytical solution for transient flow to a well near a stream that deals with clogging of the bed more directly. He assumes that a thin layer of low-hydraulic conductivity and no storativity separates the aquifer from a fully penetrating stream. As the stream is fully penetrating, the problem domain is semi-infinite; pumping on one side of the stream does not induce flow on the other side.

Various authors have attempted to analyze the errors introduced by the assumptions in these analytical models by comparison with numerical solutions of more realistic settings. Sophocleous et al. (1995) concluded that the primary limitations of commonly used analytical models are the inability to model accurately the effects of streambed clogging and the partial penetration of a stream. In a similar study, Spalding and Khaleel (1991) also suggest that neglecting the flow field on the side of the stream opposite the well can generate substantial errors.

Hunt (1999) developed an analytical solution to the problem of transient flow to a well near a partially penetrating stream with a partially clogged streambed. Hunt's solution assumes a stream of small width and a small penetration of the streambed into the aquifer. Among the analytical solutions listed here, Hunt's is unique in that it predicts flow and drawdown in the aquifer on both sides of the stream; the problem domain is infinite. Subsequently, Zlotnik and Huang (1999) and Butler et al. (2001) developed transient solutions for streams of finite width in bounded domains; analytic solutions are developed in the Laplace space, and inverted numerically. Butler et al. (2001) compared their stream depletion results with those obtained by Hunt (1999) and concluded that Hunt's results are accurate when the distance from the stream to the

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pumping well exceeds five stream widths. More recently, Fox et al. (2002) extended Hunt's solution for streams of negligible width to cases of small but finite width. Fox et al. also showed, by comparison of drawdowns, that Hunt's results are accurate for narrow streams. Apparently, the approximation of the stream made by Hunt is useful in many practical situations.

We present an explicit analytic solution for steady, two-dimensional ground water flow to a well near the leaky, or partially clogged, streambed of a partially penetrating stream. Here we deal with the stream in the same fashion as Hunt (1999): We approximate the distributed leakage across the streambed as occurring along the centerline of the stream. Our solution includes a uniform flow in the far field and a sloping hydraulic gradient of the stream. The problem domain is infinite and pumping on one side of the stream induces flow on both sides of the stream.

Transient behavior, such as that considered in studies of stream depletion, cannot be predicted with a steady model. However, steady models allow for clear visualization of flow fields through the use of the stream function; some flow phenomena can be understood more clearly when steady conditions are considered. In addition, steady models are useful for distinguishing between flow regimes and for capture zone analyses. We investigate flow regimes and the effects of parameters on the capture zone for wells pumping near streams. In particular, we examine the exchange of water between the stream and aquifer, the behavior of the flow field on the side of the stream opposite from the pumping well, and the effects of the conditions on the side opposite the well on the capture zone of the well.

Statement of Problem

We consider steady, two-dimensional ground water flow to a well near a stream. The problem consists of two domains, D_1 and D_2, in the complex z-plane as shown in Figure 1; D_1 is the lower half plane ($y \leq 0$), D_2 is the upper half plane ($y \geq 0$). We define a complex potential Ω as

$$\Omega = \Phi + i\Psi \tag{1}$$

where Φ [L^3/T] is the discharge potential and Ψ [L^3/T] is the stream function. Φ is defined, for confined ground water flow or for linearized, unconfined flow, as

$$\Phi = Th \tag{2}$$

where T [L^2/T] is the transmissivity of the aquifer, and h [L] is the piezometric head. The complex potential is an analytic function of the complex coordinate $z = x + iy$. The complex discharge function $W(z)$ is defined as

$$W = -\frac{d\Omega}{dz} = Q_x - iQ_y \tag{3}$$

where Q_x and Q_y [L^2/T] are the x and y components of the comprehensive discharge vector (the vertically integrated specific discharge vector).

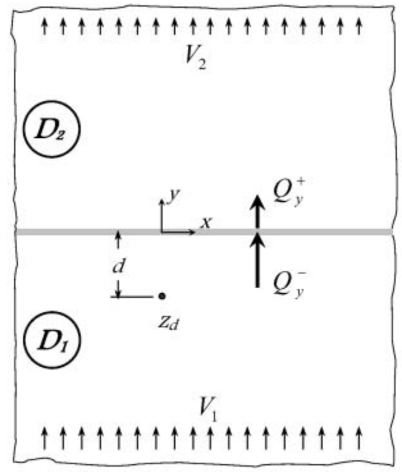

Figure 1. Definition sketch: flow to a well near a leaky stream bed.

A well of discharge Q [L^3/T] is located at $z = z_d$ in D_1; the well is illustrated in Figure 1 as the black dot at a distance d from the stream ($z_d = -id$). A stream of width $2B$ and with a partially clogged streambed lies along the real axis. The stream is represented in the figure as the thick gray line. The hydraulic head of the stream, h^*, varies linearly with x:

$$h^* = ux + h_0 \tag{4}$$

where u is a constant slope and h_0 is the hydraulic head of the stream at $x = 0$. The stream penetrates the aquifer only slightly, allowing water to flow under the stream from D_2 to D_1 or vice versa. This behavior is illustrated in Figure 2a, which shows pathlines in a vertical section of the aquifer taken along the y-axis. Note that the flow is chosen toward the stream for $y > 0$. The pumping well induces flow from the stream to the aquifer, and causes ground water from the opposite side of the stream to flow beneath the stream.

The vertical leakage through the streambed is dependent on the head difference between the aquifer and the stream, and the resistivity of the streambed. The vertical leakage γ [L/T] from the aquifer through the streambed into the stream is given by

$$\gamma = \frac{h - h^*}{c} \tag{5}$$

where h is the head in the aquifer beneath the stream, and c [T] is the vertical resistance, or resistivity, of the streambed; the resistivity is the reciprocal of the vertical leakance. We approximate the net flow into the river, σ [L^2/T], as the leakage at the center of the stream multiplied by the width of the stream:

$$\sigma = 2B\gamma = \frac{2B}{c}[h(x,0) - h^*(x)] \tag{6}$$

or

$$\sigma = \frac{2B}{cT}[\Phi(x,0) - \Phi^*(x)] \tag{7}$$

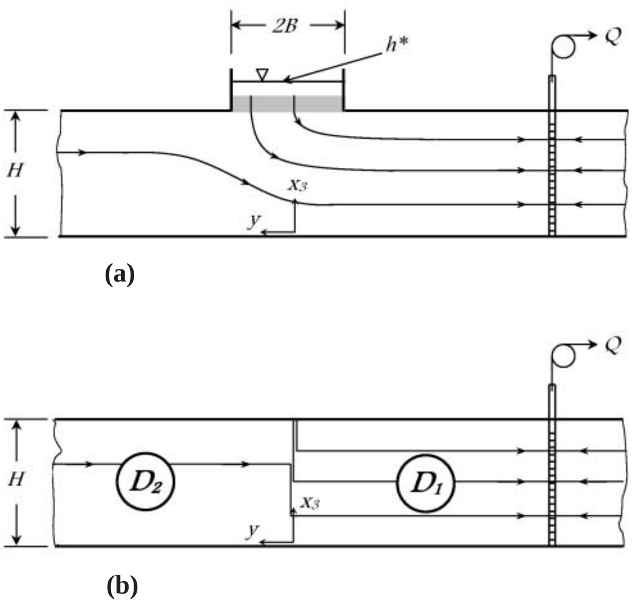

Figure 2. Pathlines in a vertical cross section through the stream and the well: (a) the exact behavior caused by distributed leakage, and (b) the approximate behavior when leakage is lumped at the center of the stream.

where

$$\Phi^*(x) = Th^*(x) = -Ux + \Phi_0 \tag{8}$$

and

$$U = -Tu \qquad \Phi_0 = Th_0 \tag{9}$$

We have neglected the effects of distributed leakage through the streambed by lumping the effects along the real axis. We may write a boundary condition on the two domains relating the jump in the component of flow normal to the stream and the net inflow into the stream:

$$Q_y^- - Q_y^+ = \sigma = \frac{2B}{cT}[\Phi(x,0) - \Phi^*(x)] \tag{10}$$

The notation with the superscripts + and – is used throughout this paper and indicates evaluation along the stream (the real axis), just inside domains D_2 and D_1, respectively. Furthermore, we require the heads, and thus the potentials, in the two domains to be continuous along the real axis:

$$\Phi^+ = \Phi^- \tag{11}$$

The behavior of pathlines in our approximate model is shown in Figure 2b for the same vertical section as shown in Figure 2a. This behavior is discussed by Bakker and Strack (1996) with respect to capture zone analysis near partially penetrating ditches.

Finally, we specify the flow at infinity to be uniform in the y direction, but with different values in the two domains:

$$Q_y = V_1 \qquad z \to \infty \text{ in } D_1 \tag{12}$$

$$Q_y = V_2 \qquad z \to \infty \text{ in } D_2 \tag{13}$$

The x component of flow at infinity in each domain is determined as part of the solution.

The model described here is intended to represent local conditions. For a stream with a sloping hydraulic head, the head of the stream will eventually fall below the bottom of the aquifer. This is a practical limitation, however, and not a mathematical one; the problem is well posed and the solution is presented in the next section.

Solution

The solution to this problem may be obtained by application of the method of images for leaky boundaries described by Anderson (2000). Here we skip the derivation, present the solution, and demonstrate that the solution satisfies all boundary conditions. The solution is

$$\Omega = \frac{Q}{2\pi}\ln\frac{z - z_d}{z - \overline{z_d}} - \frac{Q}{2\pi}\exp(Z_1)\mathrm{E}_1(Z_1) - (U - iV_1)z + A \qquad y \le 0 \tag{14}$$

$$\Omega = -\frac{Q}{2\pi}\exp(Z_2)\,\mathrm{E}_1(Z_2) - (U - iV_2)z + A \qquad y \ge 0 \tag{15}$$

where

$$Z_1 = \frac{iB}{cT}(z - \overline{z_d}) \tag{16}$$

$$Z_2 = -\frac{iB}{cT}(z - z_d) \tag{17}$$

where an overbar denotes the complex conjugate, and where A is a real constant given by

$$A = \frac{cT}{2B}(V_1 - V_2) + \Phi_0 \tag{18}$$

$\mathrm{E}_1(z)$ is the exponential integral of z, as defined by Abramowitz and Stegun (1965):

$$\mathrm{E}_1(z) = \int_z^\infty \frac{\exp(-t)}{t}dt \tag{19}$$

The complex discharge function is obtained from differentiation of Ω, and, after combining terms, gives

$$W = -\frac{Q}{2\pi}\left[\frac{1}{z - z_d} - \frac{iB}{cT}\exp(Z_1)\mathrm{E}_1(Z_1)\right] + U - iV_1 \qquad y \le 0 \tag{20}$$

$$W = -\frac{Q}{2\pi}\left[\frac{1}{z - z_d} + \frac{iB}{cT}\exp(Z_2)\mathrm{E}_1(Z_2)\right] + U - iV_2 \qquad y \geq 0 \tag{21}$$

We demonstrate that the solution (Equations 14 and 15) is correct by checking the boundary conditions. First, we consider continuity of the potential across the stream (Equation 11). We evaluate the discharge potentials (the real part of Ω) along the real axis. For $y = 0$, $|z - z_d| = |z - \overline{z_d}|$ and the real part of the logarithmic term in Equation 14 vanishes. Therefore, we obtain

$$\Phi^- = -\frac{Q}{2\pi}\Re\left[\exp(Z_1^-)\mathrm{E}_1(Z_1^-)\right] - Ux + A \tag{22}$$

$$\Phi^+ = -\frac{Q}{2\pi}\Re\left[\exp(Z_2^+)\mathrm{E}_1(Z_2^+)\right] - Ux + A \tag{23}$$

From symmetry it is known that (Abramowitz and Stegun 1965)

$$\Re\left[\exp(z)\mathrm{E}_1(z)\right] = \Re\left[\exp(\bar{z})E_1(\bar{z})\right] \tag{24}$$

Along the stream $Z_2 = \overline{Z_1}$, so that the symmetry relationship may be applied to give

$$\Re\left[\exp(Z_2^+)\mathrm{E}_1(Z_2^+)\right] = \Re\left[\exp(Z_1^-)E_1(Z_1^-)\right] \tag{25}$$

Application of Equation 25 to Equations 22 and 23 shows that the boundary condition (Equation 11) is satisfied exactly.

Next, we consider the second boundary condition (Equation 10). The y-component of the discharge vector in each domain, using Equations 3, 20, and 21, is

$$Q_y = \frac{Q}{2\pi}\Im\left[\frac{1}{z - z_d} - \frac{iB}{cT}\exp(Z_1)\mathrm{E}_1(Z_1)\right] + V_1 \qquad y \leq 0 \tag{26}$$

$$Q_y = \frac{Q}{2\pi}\Im\left[\frac{1}{z - z_d} + \frac{iB}{cT}\exp(Z_2)\mathrm{E}_1(Z_2)\right] + V_2 \qquad y \geq 0 \tag{27}$$

We evaluate the jump in Q_y along the real axis, using Equations 26 and 27, and Equation 25, to obtain

$$Q_y^- - Q_y^+ = -\frac{QB}{cT\pi}\Re\left[\exp(Z_1^-)\mathrm{E}_1(Z_1^-)\right] + V_1 - V_2 \tag{28}$$

The difference in the potential below the stream and in the stream may be evaluated from Equation 22 with Equations 18 and 8:

$$\Phi(x, 0) - \Phi^*(x) = -\frac{Q}{2\pi}\Re\left[\exp(Z_1^-)\mathrm{E}_1(Z_1^-)\right] + \frac{cT}{2B}(V_1 - V_2) \tag{29}$$

Substitution of Equations 28 and 29 in Equation 10 shows that the boundary condition (Equation 10) is also satisfied exactly.

Finally, we must evaluate the behavior of the potential in each domain at infinity. We consider the behavior of the term $\exp(z)\mathrm{E}_1(z)$ for z approaching infinity. From Abramowitz and Stegun (1965) we have

$$\lim_{z \to \infty} \exp(z)\mathrm{E}_1(z) = 0 \tag{30}$$

Since $1/(z–z_d)$ also approaches zero at infinity, Q_y (Equations 26 and 27) approaches V_1 at infinity in D_1 and V_2 in D_2. The behavior at infinity in both domains is correct. As such, all boundary conditions are met and we have verified that Equations 14 and 15 are the solution to the stated problem.

We may now evaluate the x component of flow, Q_x, at infinity in the two domains; we find from Equations 20 and 21 that the x component of flow at infinity is the same in both domains, and is equal to U (Equation 9). Hence, the gradient of the stream defines the x component of flow at infinity.

The presented solution is applicable to a well at an arbitrary location z_d. Solutions with multiple wells may be obtained through superposition. In the remaining part of this paper we will consider flow to a single well located at $z_d = –id$. An example flow field with one well is shown in Figure 3. The dashed lines are contours of piezometric head, and the solid lines are contours of the stream function. The dimensionless parameters for this example are $Bd/(cT) = 0.4$, $V_1\pi d/Q = 0.5$, $V_2/V_1 = –0.3$, and $U/V_1 = –0.5$. It may be verified visually from Figure 3 that the head is continuous across the stream (Equation 11). It is also clear from the figure that the normal component of flow jumps across the stream, but the magnitude of the jump cannot be interpreted easily from the figure.

Discharge from the Stream

Along the stream, the y component of flow may be written in dimensionless form, using Equations 26 and 27 with $z_d = -id$, as

$$\frac{Q_y^- d}{Q} = -\frac{1}{2\pi}\left[\frac{1}{(x/d)^2 + 1} + \frac{Bd}{cT}\Re\left[\exp(Z_1^-)\mathrm{E}_1(Z_1^-)\right]\right] + \frac{V_1 d}{Q} \tag{31}$$

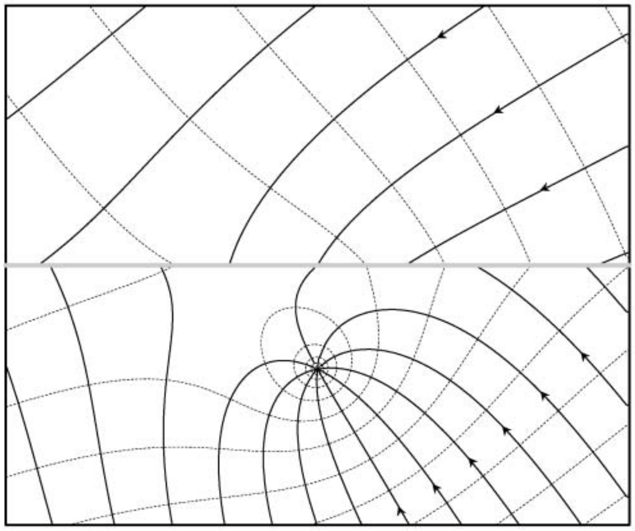

Figure 3. Flow field for *Bd/(cT)* = 0.4, V_2/V_1 = –0.3, and U/V_1 = –0.5.

$$\frac{Q_y^+ d}{Q} = -\frac{1}{2\pi}\left[\frac{1}{(x/d)^2+1} - \frac{Bd}{cT}\Re[\exp(Z_2^+)\mathrm{E}_1(Z_2^+)]\right] + \frac{V_2 d}{Q} \tag{32}$$

The net flow σ into the stream may also be written in dimensionless form, using Equation 10, as

$$\frac{\sigma d}{Q} = \frac{1}{\pi}\frac{Bd}{cT}\Re\left[\exp(Z_1^-)\mathrm{E}_1(Z_1^-)\right] + \frac{(V_1 - V_2)d}{Q} \tag{33}$$

The net flow σ into the stream may be divided into two parts: the first part of Equation 33 represents the interaction between the well and the stream, the second part represents the inflow due to the difference in the uniform gradients V_1 and V_2. Note that σ is independent of the gradient *u* of the water level in the stream. The interaction between the well and the stream is evaluated by setting $V_1 = V_2 = 0$. The variation along the stream of the component of flow normal to the stream in D_2 (Q_y^+) and in D_1 (Q_y^-) are shown in Figure 4a. The net flow from the stream to the aquifer (i.e., –σ) is also shown in Figure 4a. Figure 4a represents the case that $Bd/(cT) = 0.4$; the solid line is $Q_y^- d/Q$, the dashed line $Q_y^+ d/Q$, and the bold line represents $-\sigma d/Q$, the dimensionless discharge from the stream to the aquifer.

When the slope *u* is also set to zero, we arrive at the special case of flow to a well near a stream with a constant hydraulic head and with no flow from infinity; all water pumped from the well comes from the stream. This is the steady-state, complex equivalent of the transient solution of Hunt (1999); a real-valued expression for the drawdowns of this special case is also found in Hunt (2003). The flow field for the case $Bd/(cT) = 0.4$ is shown in Figure 5.

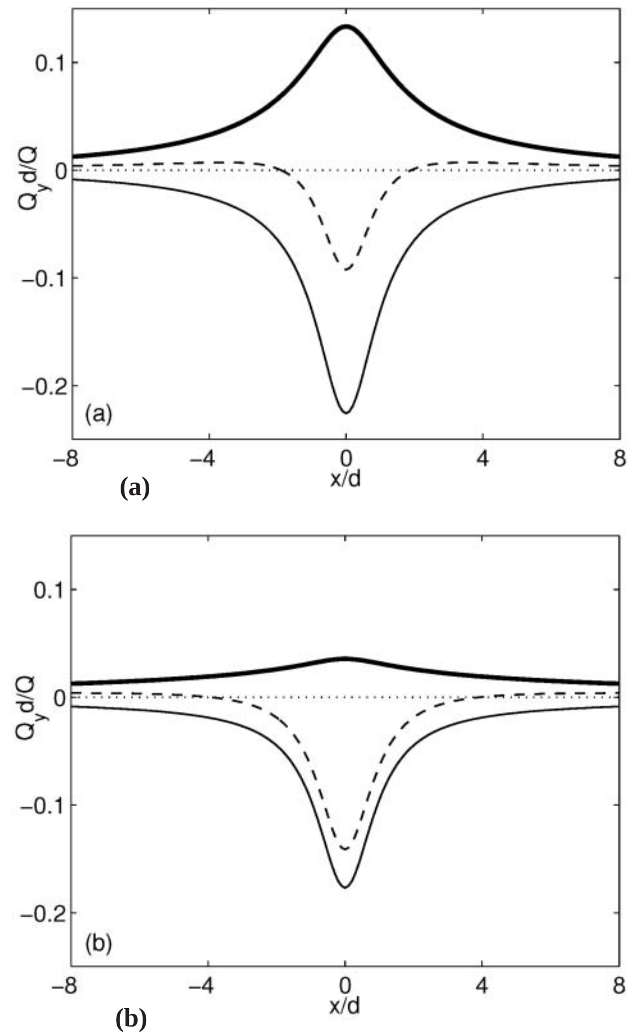

Figure 4. Dimensionless discharge versus dimensionless distance along stream for (a) *Bd* = *(cT)* = 0.4, and (b) *Bd/(cT)* = 0.04; $Q_y^- d/Q$ (dashed), and $-\sigma d/Q$ (bold).

The flow on the side of the stream opposite the pumping well is of particular interest; the pumping well produces a circulating flow pattern in D_2. Because there is no flow at infinity for this case, the net discharge in D_2 is zero, while in D_1, the side of the stream with the well, the net discharge is *Q*.

Along the center section of the stream in D_2, water flows beneath the stream to the well ($Q_y^+ < 0$, Figure 4); on either side of the center section, water flows from the stream into the aquifer ($Q_y^+ > 0$, Figure 4). The length of the center section along which flow is towards the stream in D_2 is ~1.9*d* for $Bd/(cT) = 0.4$ (Figure 4a). When the resistance of the streambed increases, the length of this section increases. In addition, the length of stream that contributes significant amounts of water to the well also increases, as may be seen from the bold line in Figure 4b, where $Bd/(cT) = 0.04$.

Capture Zones for Wells Near Streams

We consider flow regimes and capture zone envelopes for the case that the stream has a constant hydraulic head (*u* = 0) and flow is toward the stream in D_1 ($V_1 > 0$). In this case, there are two possible flow regimes, analogous to the regimes for flow to a well near a fully penetrating river

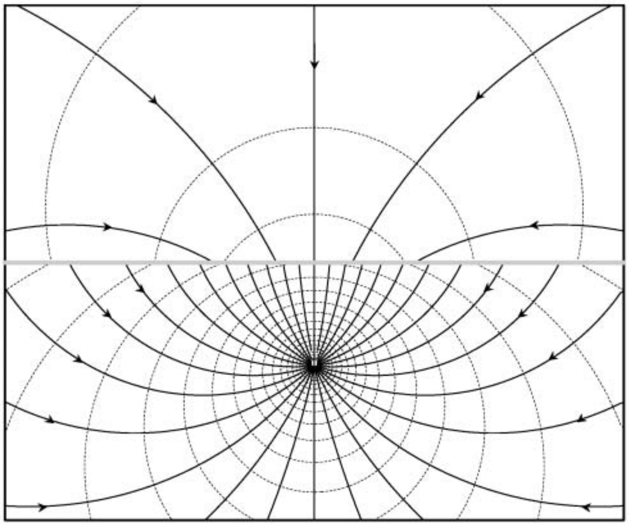

Figure 5. Flow field for the special case $U = V_1 = V_2 = 0$, with $Bd/(cT) = 0.4$.

without resistance (Strack 1989, pp. 43–46); the well either captures river water or does not. Flow fields and dividing streamlines in D_1, representative of the two regimes, along with the transition case are shown in Figure 6; in all three cases, the parameters are $Bd/(cT) = 0.4$ and $V_2 = 0$. Figure 6a shows an example of regime 1 flow, where $V_1\pi d/Q = 1$ and no river water is captured by the well; Figure 6b shows the transition case separating regimes 1 and 2, where $V_1\pi d/Q = 0.7096$; Figure 6c shows an example of regime 2 flow, where $V_1\pi d/Q = 0.5$ and the capture zone envelope for the well intersects the river and river water is captured by the well.

We investigate the conditions for which the transition case, illustrated in Figure 6b, occurs. Under transition conditions, the well does not capture any water from the river, or from the other side of the river; the y-component of the discharge vector in D_1 must be positive along the entire real axis, with the critical point being at the origin (see also Figure 4). A condition for the transition case may be obtained by setting Q_y in D_1 equal to 0 at the origin:

$$Q_y^-(x = 0) = 0 \tag{34}$$

Setting Equation 31 equal to zero at $x = 0$, using $Z_1^-(x = 0) = Bd/(cT)$, gives

$$\frac{V_1\pi d}{Q} = \frac{1}{2} + \frac{Bd}{2cT}\exp\left(\frac{Bd}{cT}\right)\mathrm{E}_1\left(\frac{Bd}{cT}\right) \tag{35}$$

We see from Equation 35 that the transition between regimes is independent of the uniform flow V_2 specified in D_2. The relationship between parameters given by Equation 35 is shown graphically in Figure 7; points lying above the transition line are representative of regime 1 flow and points lying below the line are representative of regime 2 flow. It is important to note that the transition line defined by Equation 35 is approximate as the condition (Equation 34) does not consider directly the width of the stream.

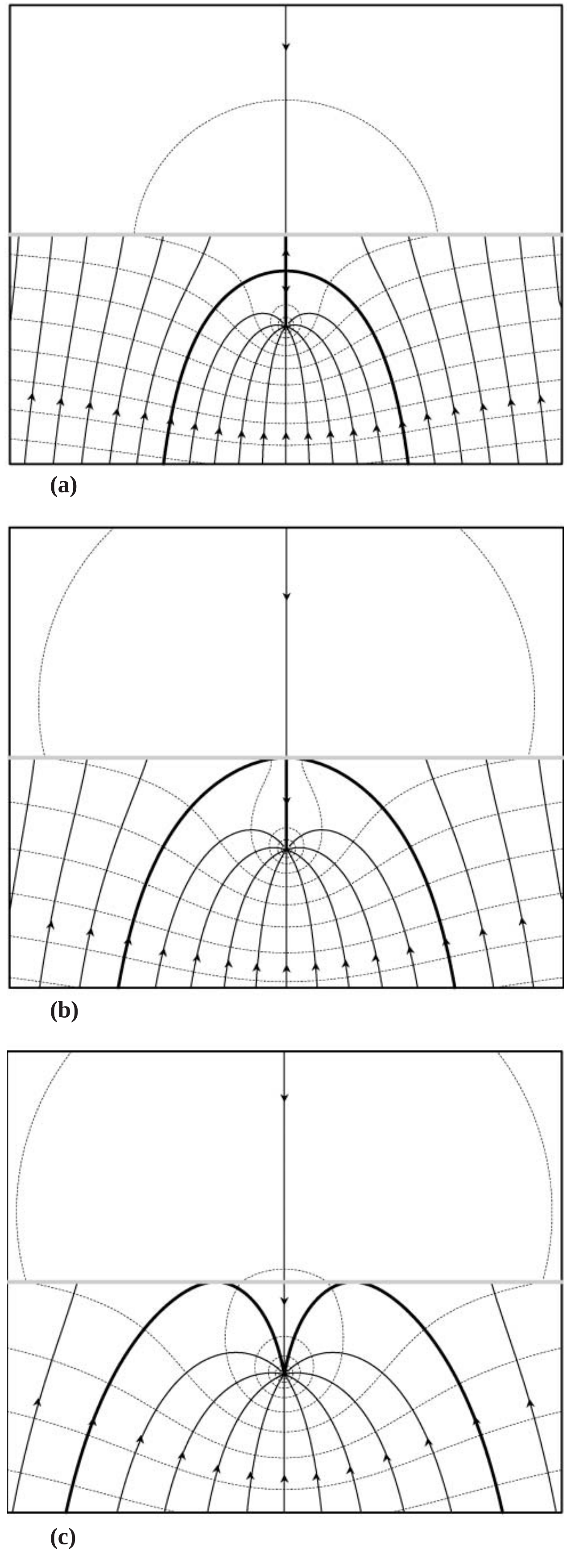

Figure 6. Examples of the two flow regimes in D_1: (a) regime 1 with no flow from the stream to the well; (b) the transition case; (c) regime 2 with flow from the stream to the well. In all cases, $Bd/(cT) = 0.4$ and $V_2 = 0$. The thick line is the dividing streamline.

There are two limiting cases of Equation 35 that may be examined. First, when there is no resistance layer at the bottom of the stream ($c = 0$), the condition reduces to $V_1\pi d/Q = 1$ as

$$\lim_{z\to\infty} z\exp(z)\mathrm{E}_1(z) = 1 \tag{36}$$

This solution may be obtained by the classic method of images (Strack 1989, pp. 43–46). Second, when the bottom of the stream is impermeable ($c = \infty$), the condition becomes $V_1\pi d/Q = ½$ since

$$\lim_{z\to 0} z\exp(z)\mathrm{E}_1(z) = 0 \tag{37}$$

This case is equivalent to a well in uniform flow (without the presence of a stream) with a stagnation point a distance d downstream of the well (Haitjema 1995, p. 58). Equations 36 and 37 may be verified with standard methods.

When the hydraulic head in the stream varies linearly with x (i.e., $u \neq 0$), the condition $Q_y^- \geq 0$ is still satisfied by Equation 35; the well captures no river water. In this case, however, the capture zone envelope for the well does not touch the stream, and Equation 35 does not provide the transition case between regimes 1 and 2. This may be seen from Figure 8, which shows the flow field obtained when conditions in D_1 are identical to those given for Figure 6b, but with $U/V_1 = -1$. We conclude that the condition (Equation 35) is conservative for streams with a sloping hydraulic head; if the condition is maintained, the well will capture no water from the stream or from the other side of the stream (a similar result was obtained by Newsom and Wilson [1988] for a stream in full contact with the aquifer).

Next, we consider the effects of the flow field in D_2 on the capture zone envelope of the pumping well. We have shown already that the behavior of the flow field in D_2 has no effect on the transition condition (Equation 35) and therefore does not affect the shape of the capture zone envelope for regime 1 flow. However, the conditions in D_2 do have an impact on the capture zone envelope under regime 2 conditions. This is demonstrated in Figures 9a and 9b. In both figures, the flow conditions in D_1 are the same as those given for Figure 6c. In D_2, however, we vary the flow conditions: in Figure 9a, $V_2/V_1 = -0.3$ and thus flow in D_2 is toward the stream, while in Figure 9b, $V_2/V_1 = 0.3$ and flow in D_2 is away from the stream. In the figures, the shaded region represents the capture zone envelope for the well and the dashed-dotted line represents the streamline in D_1 that just touches the stream.

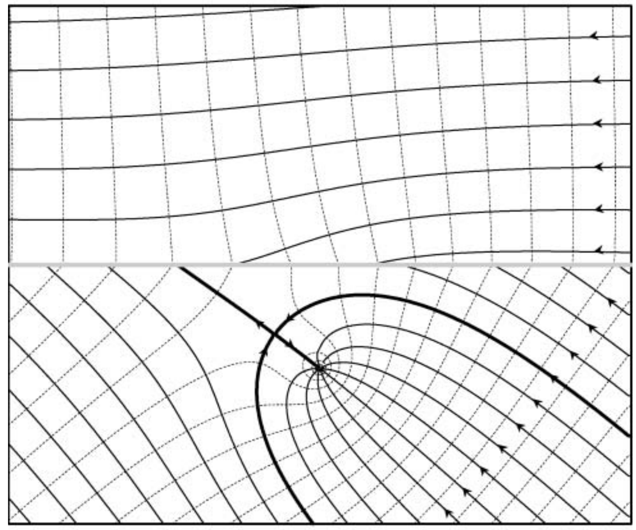

Figure 8. Capture zone envelope near a stream with a sloping hydraulic head showing the effects of the stream gradient on transition conditions.

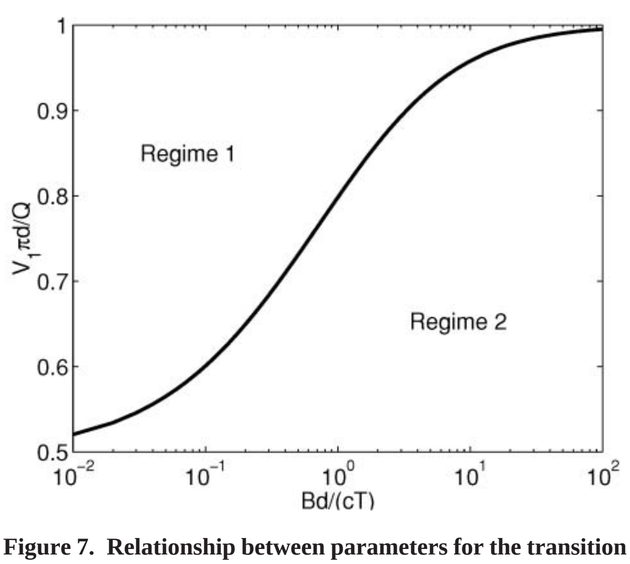

Figure 7. Relationship between parameters for the transition case. Regime 1 means no river water is captured by the well. Regime 2 means river water is captured by the well.

The capture zone envelope shown in Figure 9a includes a wide region in D_2 extending to infinity, from which water may be drawn beneath the stream and captured by the well. In D_1, the capture zone envelope is bounded by the dashed-dotted streamline. In Figure 9b the capture zone envelope for the well also extends to the other side of the stream, but the behavior there is different; there is a finite region from which water may be captured by the well. A comparison of the capture zone envelopes in D_1 in Figures 9a and 9b shows that the envelope in the latter extends beyond the dashed-dotted streamlines; the capture zone envelope is wider in D_1 in the latter case, even though the flow conditions in D_1 are identical.

We conclude that in cases of regime 2 flow, the size of the capture zone in D_1 is dependent on conditions in D_2; conditions on both sides of the stream must be identified to evaluate the capture zone of the pumping well properly. In this respect, it is noted that the uniform flows V_1 and V_2 represent the uniform flows normal to the stream before the well is pumped.

Discussion and Conclusions

We have presented an analytic solution, in the form of a complex potential, for steady-state flow to a well near a stream with a leaky or partially clogged streambed; the solution allows for a different uniform gradient toward the river on either side and a sloping hydraulic head in the stream. The solution is valid for narrow streams, for which the leakage through the streambed may be approximated as

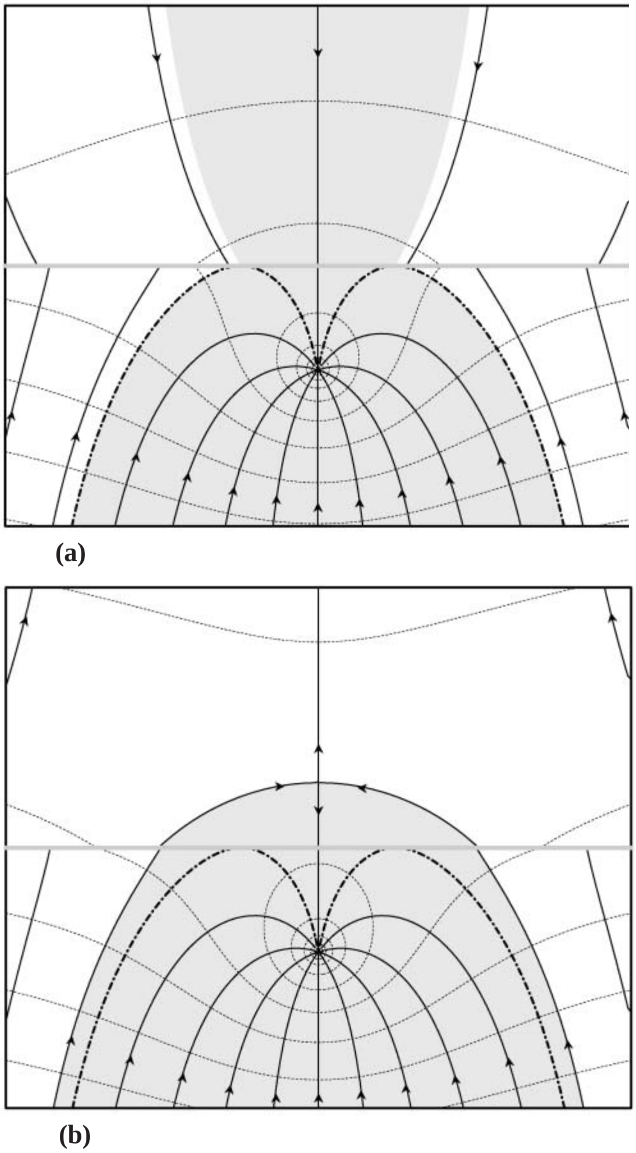

Figure 9. Dependence of the capture zone in D_1 on flow conditions in D_2. (a) The case $V_2/V_1 = -0.3$, and (b) the case $V_2/V_1 = 0.3$. In both cases $Bd/(cT) = 0.4$, and $V_1 \pi d/Q = 0.5$.

occurring along the centerline of the stream. Equations were presented for the complex potential (and thus the head and stream function) as a function of *x* and *y* on both sides of the stream; the well causes a drawdown on the side of the stream opposite the pumping well for any finite value of the vertical resistance of the streambed.

For the case that both uniform gradients and the slope of the head in the stream are zero, the solution reduces to the steady-state, complex equivalent of the transient problem solved by Hunt (1999). Hunt's transient solution is also valid for the case that the uniform gradients are not zero, as the uniform gradients may be superimposed. The solution provides some insight in the flow pattern on the side of the stream opposite the well. Far away from the well, the stream acts as a source, while nearby water flows beneath the stream towards the well.

Two applications of a steady-state solution are the evaluation of flow regimes, and the delineation of capture zones and capture zone envelopes. Flow regimes were evaluated analytically and visualized by drawing contour plots of heads and stream function. We have presented conditions for which the capture zone of the well does not intersect the stream (regime 1). It was shown that if the capture zone envelope intersects the stream (regime 2), then it will always extend beyond the stream. Also, it was shown that for regime 2 flow, the uniform gradient on the side of the stream opposite the well has an effect on the shape of the capture zone on the side of the stream with the well.

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